Journal of Mathematical Systems, Estimation, and Control
Vol. 6, No. 1, 1996, pp. 1–12

Asymptotic Stabilization of a Class of Three Dimensional Homogeneous Control Systems*

Sandra Lee Samelson

Abstract

We consider the stabilization problem for three dimensional homogeneous polynomial systems, and derive an intrinsic sufficient condition for stablizability. We show that this condition is satisfied by an open set in the space of homogeneous systems of a fixed degree.

Key words: asymptotic stabilization, nonlinear control, homogeneous control systems

AMS Subject Classifications: 93C10, 93B17, 93B18

1 Introduction

Asymptotic stabilization of nonlinear control systems has been a subject of active research over the past several years. The major motivating factor has been the realization that existing theories on control systems analysis and design are inadequate for solving modern day problems in robotics, advanced aircraft, smart structures, and a variety of complex nonlinear systems. In order to solve these complicated problems, more and more advanced mathematical tools have begun to be employed, thus leading way to the development of a rich theory of nonlinear control systems analysis.

One of the classes of nonlinear systems that display a structure, potentially rich enough to enable a full understanding of the stabilizability aspects, is that of homogeneous polynomial systems. These have the general structure,

$$\dot{x} = f(x) + \sum_{i=1}^{m} b_i u_i;$$
$$x \in \Re^n, u_i \in \Re, b_i \in \Re^n, \tag{1.1}$$

*Received January 1, 1994; received in final form January 13, 1995. Summary appeared in Volume 6, Number 1, 1996.

where f is a vector field, which, when written using the standard coordinates of $\Re^n$, has the property that all entries are homogeneous polynomials of some degree $p > 0$. The case when $p = 1$ is the linear case, which has been completely understood for well over three decades. The first nontrivial class of such systems arises when $m = 1, n = 3$, and $p = 2$, i.e. three dimensional, quadratic, homogeneous systems. The stabilizability of this class was considered and resolved for a generic subset in [6]. Here we address the case $m = 1, n = 3$, and $p \geq 2$. We consider a system of the form,

$$\dot{x} = f(x) + bu, \; x \in \Re^3, \; b \in \Re^3, \; u \in \Re, \tag{1.2}$$

where $f(x)$ is a homogeneous polynomial vector field, and b is a constant vector field. The problem considered here is under what conditions on f and b can one construct a positively homogeneous feedback function $u = \alpha(x)$ of degree p which will asymptotically stabilize (1.2).

In addition to the rich mathematical structure that motivates the problem, one can consider this as a stabilization problem for systems with a p^{th} order singularity at an equilibrium. Vector fields f and b are the p^{th} order and the zeroth order jets of the state and the input vector fields. We will illustrate this aspect by considering an example of a system undergoing Hopf/Hopf/Stationary bifurcation.

2 Preliminaries on Stabilization of Three-Dimensional Homogeneous Systems

In this section we consider a system,

$$\dot{z} = F(x) + bu, \; x \in \Re^3, \; b \in \Re^3, \; u \in \Re, \tag{2.1}$$

where F is a homogeneous vector field of some degree $p > 0$. Here, we use the term homogeneity to indicate homogeneity along positive rays i.e. $F(\lambda(x)) = \lambda^p(F(x))$ for all $x \in \Re^n$, for all $\lambda > 0$. Here we review some important concepts that will play a key role in the analysis of our problem in the subsequent chapters.

First, we will assume without any loss of generality, that $b = [0, 0, 1]^T$. This can be ensured via a linear coordinate transformation in $\Re^3$. Now, we can write (2.1) in the form,

$$\begin{aligned} \dot{x} &= f(x, y) \\ \dot{y} &= u \end{aligned} \tag{2.2}$$

where, $x \in \Re^2$, $y \in \Re$, $u \in \Re$, and f is positively homogeneous of degree p. Pioneering work by Coleman (see [1]) has shown that the most promising method of analyzing such systems is to first focus on the radial projection of (2.2) onto the unit sphere in $\Re^3$. This has the advantage of cutting down the dimensionality of the state-space from three to two, and well-known theorems due to Poincare and Bendixon, etc. give a complete description of the phase-portrait of a dynamical system living on the two-dimensional sphere.

Coleman's theorem states that asymptotic stability of a positively homogeneous three dimensional dynamical system on $\Re^3$ is determined completely by the stability properties of the restriction of the system to the cases generated by equilibria and periodic orbits of the projected system to the unit sphere. Let us first focus on equilibria. The set of points on S^2 that could be rendered equilibria of the projected dynamics of (2.2) via feedback are exactly those at which $[f(x,y)^T, 0]^T$ is parallel to the $[x^T, 0]^T$ direction. Stability on cones generated by such equilibria is determined in a rather straight-forward manner: stable if and only if $f(x,y)$ and x point in the opposite directions, and unstable if and only if they point in same direction.

Following [6] we define these crucial sets as follows: For arbitrary $\delta \in \Re$ let $\hat{A}_\delta = \{(x,y) \in \Re^{n+1} | f(x,y) = \delta x\}$. Let $A_\delta = \hat{A}_\delta \cap S^2$. Let,

$$\begin{aligned} A_+ &= \cup_{\delta>0} A_\delta, \\ A_- &= \cup_{\delta<0} A_\delta, \\ A_{+0} &= A_+ \cup A_0 \\ A_\Re &= \cup_{\delta\in\Re} A_\delta. \end{aligned} \tag{2.3}$$

Let,

$$\begin{aligned} \Omega = \{&\sigma : S^1 \to S^2 - poles \mid \sigma \text{ is a } C^1 \text{ embedding, and} \\ &\sigma \text{ is transversal to the meridians}\} \end{aligned}$$

The following lemmas were proved in [6]. Some results closely related to these can be found in [4] and [5].

Lemma 2.1 (6) *Suppose that the degree of homogeneity of (2.2) is not less than two. If there is $\sigma \in \Omega$ such that $\sigma \cap A_{+0} = \emptyset$ and $\sigma \cap A_- \neq \emptyset$, then the system is asymptotically stabilizable by C^1 positively homogeneous feedback.*

Lemma 2.2 *[6] Suppose that there exists a continuous curve $\mu : [0,1] \to S^2$ such that,*

(i) $\mu(0) =$ north pole,

(ii) $\mu(1) =$ south pole,

(iii) $\mu \subset A_{+0}$.

Then the system (2.2) does not admit a continuous positively homogeneous asymptotically stabilizing feedback function.

In this paper we will use these two lemmas in order to analyze the asymptotic stabilization problem for homogeneous cubic systems in three dimensions.

3 Stabilization of Homogeneous Systems

Here we will consider the system,

$$\begin{aligned} \dot{x_1} &= f_1(x,y) = \sum_{j=1}^{p+1} a_{1,j}(x) y^{p+1-j} \\ \dot{x_2} &= f_2(x,y) = \sum_{j=1}^{p+1} a_{2,j}(x) y^{p+1-j} \\ \dot{y} &= u, \end{aligned} \tag{3.1}$$

where $x = (x_1, x_2) \in \Re^2, y, u \in \Re, f_1$ and f_2 are homogeneous degree p polynomials. We will denote (f_1, f_2) by f.

In general, the primary motivation for studying the stabilizability of higher order systems arises due to the need to solve stabilization problems for systems undergoing bifurcation. Typically, the symmetries of the system dictate the number of critical modes, and within such limitations it is only necessary to study systems with low codimension. In view of this aspect we will impose certain genericity hypothesis on f, primarily among them $(a_{1,1}, a_{2,1}) \neq 0$. Following [6] we will first carry out some simplifications in the structure.

The structure of our system is,

$$\begin{aligned} \dot{x_1} &= a_{1,1} y^p + a_{1,2}(x) y^{p-1} + \cdots + a_{1,p+1}(x) \\ \dot{x_2} &= a_{2,1} y^p + a_{2,2}(x) y^{p-1} + \cdots + a_{2,p+1}(x) \\ \dot{y} &= u. \end{aligned} \tag{3.2}$$

and without loss of generality we will assume that $a_{1,1} \neq 0$. Let us first redefine x_1 by dividing the original x_1 by $a_{1,1}$, and x_2 by $x_2 - (\frac{a_{2,1}}{a_{1,1}}) x_1$ in order to set $a_{1,1} = 1$ and $a_{2,1} = 0$. Now, $A_{\Re}$ (see section 2 for the definition) is computed as,

$$\begin{aligned} \lambda(x,y) &= x_2 y^3 + (x_2 a_{1,2}(x) - x_2 a_{2,2}(x)) y^2 \\ &\quad + \cdots + [x_1 a_{1,p+1}(x) - x_2 a_{2,p+1}(x)] = 0. \end{aligned}$$

$\lambda(x,y) = 0$ is a homogeneous polynomial of degree $p+1$. We will be concerned with the structure of its zeroes in $\Re P^2$. We will treat it as a p^{th} order polynomial in y with coefficients which are polynomials in x.

We will be concerned with the real branches of $\lambda(x,y) = 0$, since this set is precisely $A_{\Re}$. From the expression for $\lambda(x)$, it follows that a branch of $\lambda(x,y) = 0$ can go to infinity only if x_2 goes to zero, i.e. asymptotes occur only on the $x_2 = 0$ axis.

Our results will only be applicable for the case in which these branches do not meet each other, and the restriction of the projection on the x-plane to each branch is a diffeomorphism. This is precisely the situation in which $\lambda(x,y) = 0$ has no multiple solutions at any x, which in turn is equivalent to saying that the discriminant of $\lambda(x,y)$ is definite.

3.1 Case of systems of degree three

For the sake of illustration we will first consider the case $p = 3$. This case already illustrates the primary ideas and concepts in the analysis carried out here. These ideas will be generalized to the case of degree p systems in the next subsection.

Lemma 3.1 *(see e.g. [9]) Consider a cubic polynomial equation with real coefficients, $z^3 + cz + d = 0$. Its discriminant is $4c^3+27d^2$. It will have one or three real zeroes depending upon whether its discriminant is greater or less than zero.*

In our problem ($p = 3$), in order to gain some understanding of the structure of the zeroes of $\lambda(x,y)$, let us first put λ into the standard form in the lemma by redefining y by $[y + (x_2a_{1,2}(x) - x_1a_{2,2}(x))/(3x_2)]$ to get $A_{\Re}$ as the zero set of $x_2[y^3 + p(x)y + q(x)]$, where

$$\begin{aligned} c(x) &= (x_2a_{1,3} - x_1a_{2,3})/x_2 - 1/3x_2^2(x_2a_{1,2} - x_1a_{2,2})^2, \\ d(x) &= (x_2a_{1,4} - x_1a_{2,4})/x_2 \\ &\quad - ((x_2a_{1,2} - x_1a_{2,2})(x_2a_{1,3} - x_1a_{2,3}))/(3x_2^2) \\ &\quad + (2/27x_2^3)(x_2a_{1,2} - x_1a_{2,2})^3. \end{aligned} \tag{3.3}$$

Let us rewrite $c(x)$ and $d(x)$ as,

$$\begin{aligned} c(x) &= \bar{c}(x)/x_2^2, \\ d(x) &= \bar{d}(x)/(x_2)^3, \end{aligned}$$

where $\bar{c}(x)$ and $\bar{d}(x)$ are homogeneous polynomials of degrees four and six respectively. Now, the discriminant of $\hat{\lambda}(x,y)$ is

$$\begin{aligned} \Delta(x) &= 4(c(x))^3 + 27(d(x))^3 \\ &= [4(\bar{c}(x))^3 + 27(\bar{d}(x))^2]/x_2^6. \end{aligned}$$

After simplification it can be observed that for a generic set of coefficients, x_2^2 can be factored out from $4(\bar{c}(x))^3 \; + \; 27(\bar{d}(x))^2$. Hence we can write,

$$\Delta(x) \; = \; r(x)/x_2^4$$

where $r(x)$ is a tenth degree polynomial and x_2 isn't a factor of $r(x)$ for generic values of $a_{i,j}, i = 1, 2, j = 1, \cdots, 4$. In this paper our focus is on the case in which there aren't any multiple zeroes of $\lambda(x, y) = 0$, which is the same as $r(x)$ is definite.

The obvious question we need to ask at this juncture is whether there are any cubic systems for which $r(x)$ is definite. The answer is affirmative. Let us consider the system,

$$\begin{aligned} \dot{x_1} &= y^3 - 2x_2 y^2 - (x_1^2 + x_2^2)y + 2(2x_1^2 x_2 + x_2^3) \\ \dot{x_2} &= 2x_1 y^2 - 2x_1^3 \\ \dot{y} &= u. \end{aligned} \tag{3.4}$$

For this system, $\lambda(x, y) = [x_2 y - 2(x_1^2 + x_2^2)][y^2 - (x_1^2 + x_2^2)]$. Therefore, the zeroes of λ are, $2(x_1^2 + x_2^2)/x_2$, $\sqrt{(x_1^2 + x_2^2)}$ and $-\sqrt{(x_1^2 + x_2^2)}$. Clearly, these branches don't meet each other or cross themselves. Therefore, this corresponds to a cubic system for which $r(x)$ is definite. Indeed, it can be verified that $r(x)$ is equal to $-4(3x^2 + 4y^2)^2(x^2 + y^2)^3$ which is clearly negative definite.

Since the definiteness of $r(x)$ depends algebraically on the coefficients $a_{i,j}$ we conclude that there is a nonempty open subset of the space of cubic systems for which $r(x)$ is definite.

Our main theorem, which is stated in the next section, states that when $r(x)$ is definite the system is asymptotically stabilizable. The proof of this theorem can be given without any additional complexities in the general case of degree p systems. Therefore, we will conclude the discussion of cubic systems here.

3.2 Case of systems of degree p

Now we return to the general situation of homogeneous systems of degree p. In this case we can write the discriminant in the form,

$$\Delta(x) \; = \; r(x)/x_2^n \tag{3.5}$$

where, $r(x)$ is a polynomial which does not admit x_2 as a factor for generic values of coefficients $a_{i,j}$, and n is a positive integer which depends only on p. We can modify the example 3.4 by multiplying $\lambda(x, y)$ by a positive definite homogeneous degree $p - 3$ polynomial $q(x, y)$ if p is odd, or by

$yq(x, y)$, where $q(x, y)$ is a positive definite polynomial of degree $p-4$ if p is even in order to produce an example of a degree p systems in which the real branches of $\lambda(x, y) = 0$ don't meet each other, and each projects diffeomorphically onto its image in the x-plane. Therefore, we conclude that for a given degree of homogeneity $p \geq 3$ the space of systems for which $r(x)$ is definite is a nonempty open subset. Our main theorem states that whenever $r(x)$ is definite, the system is stabilizable.

Theorem 3.1 *Consider the homogeneous polynomial system of degree p given in (3.2). Let r(x) be as in (3.5). Suppose that r(x) is definite. Then, the system is stabilizable.*

Proof: From our hypotheses branches of solutions of $\lambda(x, y) \;=\; 0$ will be distinct, and each will project to its image in the x-plane diffeomorphically. Let us focus on the $x_2 \geq 0$ half of the cylinder $\|x\|_2 = 1$. Let us number the branches of $\lambda = 0$ as $B_1, \cdots, B_k$ according to the decreasing order of their y coordinates.

Without loss of generality we will assume that $a_{2,2} \;>\; 0$. Observe that B_1 has the asymptotic description near the $x_2 \;=\; 0$ axis given by,

$$y \;=\; a_{2,2}^1 \frac{(x_1)^2}{x_2},$$

where

$$a_{2,2}^1 \;=\; (\partial/\partial x_1) a_{2,2}(x).$$

It then follows that if p is odd, points on B_1 near the (x_1, y)-plane are in A_+. Similarly, if p is even, points on B_1 near the negative x_1 axis will be in A_-. These assertions follow from the asymptotic description for the $\dot{x}_1$ equation, which is merely $\dot{x_1} \;=\; y^p$. These conclusions hold for all branches B_i which are asymptotic to the y axis.

Let B_n be the lowermost branch which is asymptotic to the y_1 axis. For the sake of simplicity we will drop the subscript n and denote it by B. Let us denote its antipodal image by C. From the ensuing discussion it follows that there is always a point z on $B \cup C$ which is in A_-. Fix z henceforth.

By our definition of n, B_i, $i > n$ are all finite branches which project diffeomorphically onto the the circle $\|x\| = 1$. Let L be a positive real number such that $|y| < L$ at all points on B_i, $i > n$, and greater than the magnitude of the y coordinate of z.

Let us parametrize the cylinder $\|x\| = 1$ using the cylindrical polar coordinate (θ, y) ($\theta = 0$ at $(1, 0)$). y coordinates of points on B_i will be smooth functions of θ, denoted here by $y \;=\; \phi_i(\theta)$.

Let $z \;=\; (\theta_0, \phi_n(\theta_0))$, $(\theta_0 \in (0, \pi))$. Let $\mu : [0, 2\pi]/\{0, 2\pi\} \to (0, 1]$ be a smooth function such that that $\mu(\theta) = 1$ iff $\theta = \theta_0$. Now define a continuous curve $\sigma : [0, 2\pi] \to$ cylinder,

$$\begin{aligned}\sigma(\theta) &= \mu(\theta)\phi_n(\theta) + (1-\mu(\theta))\phi_{n+1}(\theta);\ \text{if } \phi_n(\theta) < L \\ &= \mu(\theta)L + (1-\mu(\theta))\phi_{n+1}(\theta);\ \text{otherwise.}\end{aligned}$$

It is clear that σ meets $\lambda = 0$ only at z, and since z belongs to A_-, the hypothesis of Lemma 2.1 are satisfied. Therefore we conclude the stabilizability of the system.

Therefore, it follows that one can find a curve $\sigma : S^1 \to S^2$ satisfying the hypothesis of Lemma 1 by merely ensuring that it meets the $\lambda(x,y) = 0$ curve at a point with negative x_2 coordinate near the (x,y) - plane, and by ensuring that σ stays away from the $\lambda(x,y) = 0$ curve elsewhere. This concludes the proof of the theorem.

Remark 3.1 Observe that the only criterion used in the proof of the theorem is that the lowermost infinite branch stays above all finite branches in the $x_1 > 0$ half plane and it projects onto its image in the $\|x\| = 1$ circle diffeomorphically. Therefore, the theorem will be applicable under a wider range of hypotheses. However, it would be hard to verify these conditions without resorting to graphical means first.

However, it is still worthwhile to point out that certain popular examples satisfy these generalized hypothesis. Among them is the case of generalized integrator chains in the odd degree case. To be specific we consider the case $p = 3$.

Example 3.1

$$\begin{aligned}\dot{x_1} &= y^3 \\ \dot{x_2} &= x_1^3 \\ \dot{y} &= u.\end{aligned}$$

This example is commonly known as the cubed integrator chain of length three. Stabilizability of this system was discussed in [7], and shown that this system is stabilizable by using homogeneous polynomial cubic feedback of degree three. The techniques employed were highly specialized, and would not generalize to cover an open subset of systems.

For this system, $\lambda(x,y) = x_2 y^3 - (x_1)^4 = 0$. Therefore, $A_\Re$ consists of the curve, $y = (\frac{(x_1)^4}{x_2})^{\frac{1}{3}}$. Recall that a point on $A_\Re$ belongs to A_- if and only if $< x, f(x) > < 0$. On

$$A_\Re, < x, f(x) > = x_1 y^3 + x_1^3 x_2 = \frac{(x_1)^5}{x_2} + (x_1)^3 x_2.$$

Therefore, $A_- = \{(x_1, x_2, (\frac{(x_1)^4}{x_2})^{\frac{1}{3}}) | \frac{(x_1)^5}{x_2} + (x_1)^3 x_2 < 0\}$.

Note that in this example the $\lambda = 0$ curve has multiplicity three at all points. However, this curve projects diffeomorphically onto its image in the x-plane. Therefore, this system is stabilizable.

Observe that the discriminant for this example is,

$$\Delta(x) \;=\; 27[\frac{(-x_1)^4}{x_2}]^3 \;=\; -27\frac{(x_1)^8}{(x_2)^2}.$$

Therefore, $r(x) = (x_1)^{10}x_2^2$, which is only positive semidefinite, and hence the strict hypotheses of the theorem aren't applicable.

Example 3.2 In this example we consider a two parameter perturbation of the cubed integrator chain considered above. As far as we are aware no existing technique is applicable to determine the stabilizability of this family.

$$\begin{aligned} \dot{x_1} &= y^3 \;-\; \epsilon x_2^3 \\ \dot{x_2} &= x_1^3 + \delta y x_2^2 \\ \dot{y} &= u \end{aligned}$$

where, ϵ and δ are positive constants. We wish to find out the range of parameter values for which this system is stabilizable.

Here

$$\lambda(x_1, x_2) \;=\; x_2 y^3 - \delta x_1 x_2^2 y - (\epsilon x_2^4 + x_1^4).$$

Therefore, the discriminant is,

$$\Delta(x_1, x_2) = [27\delta^3 x_2^5 x_1^3 \;+\; (\epsilon x_2^4 + x_1^4)^2]/x_2^2.$$

Now $r(x_1, x_2) \;=\; x_2^2[27\delta^3 x_2^5 x_1^3 \;+\; (\epsilon x_2^4 + x_1^4)^2]$. (Recall that we are only interested in the $x_2 > 0$ half of the circle $x_1^2 + x_2^2 \;=\; 1$.) By using Holder's inequality it can be readily established that r is positive definite, and hence the system is stabilizable, for the range $5(27\delta^3)^{8/5}/(8(4^{3/8})) \;<\; 2\epsilon$.

Example 3.3 Here we consider a system with a Z_2 symmetry undergoing Hopf/Hopf/Stationary bifurcation. Such mode interactions occur in systems governed by partial differential equations (see e.g. [3]). Z_2 symmetries occur in nature frequently as left-right reflectional symmetries. The case of Hopf/Stationary bifurcation can be handled along similar lines, and can be reduced to a problem of stabilizing a two dimensional homogeneous system.

When a system is near a bifurcation point the linear part is weak (even though it may still be non-critical). Therefore, it makes sense to attempt to stabilize an equilibrium point of the system using higher order terms. For example, when a system is near a Hopf bifurcation point one may assume that the linear part has eigenvalues on the imaginary axis, and attempt to solve the stabilization problem using feedback to modify this linear part and quadratic, cubic and higher order terms. Normal form theory can be used to "uncouple" the linear part and the next higher order part enabling independent control of the two parts. It is this aspect we wish to explore here.

Let us consider a system,

$$\dot{x} = Ax + g(x) + bu, \tag{3.6}$$

where, A has two pairs of imaginary eigenvalues and one zero eigenvalue. The two imaginary eigenvalues are assumed to be rationally independent. Here $g(x)$ contains higher order terms. We assume that the system admits a Z_2 symmetry. This dictates that there are no even order terms in $g(x)$.

Using the normal form theory of Poincare-Birkoff (see e.g. [3]) we can use a coordinate transformation to simplify the structure of the cubic terms. After writing the two rotational modes in polar coordinates (r_1, θ_1) and (r_2, θ_2), we can write down the system of equations in the form,

$$\begin{aligned}
\dot{z} &= c_{1,1}r_1^2 z + c_{1,2}r_2^2 z + c_{1,3}z^3 + \ hot + (b_1 + \ hot)u \\
\dot{r_1} &= c_{2,1}r_1^3 + c_{2,2}r_1 z^2 + c_{2,3}r_1 r_2^2 + \ hot + (b_2 + \ hot)u \\
\dot{r_2} &= c_{3,1}r_2^3 + c_{3,2}r_2 z^2 + c_{3,3}r_2 r_1^2 + \ hot + (b_3 + hot)u \\
\dot{\theta_1} &= \omega_1 + O(|r, z|^2) + (b_4 + \ hot)u \\
\dot{\theta_2} &= \omega_2 + O(|r, z|^2) + (b_5 + \ hot)u.
\end{aligned}$$

Observe that θ_1 and θ_2 do not enter into the right-hand side of the equations at least up to order 3. Our objective here is to use feedback to stabilize the third order jet of the system.

It is clear that if we only use feedback that depends on (z, r_1, r_2) to stabilize the 3-jet of the first three equations, then that would stabilize the 3-jet of the overall system. Hence the overall problem reduces to that of stabilizing a homogeneous cubic system with a particular structure for which the theory developed in the previous section could be applied. As an example, the case,

$$\begin{aligned}
\dot{z} &= z^3 + \ hot + (1 + \ hot)u \\
\dot{r_1} &= 5r_1^3 + \ hot + (3 + \ hot)u \\
\dot{r_2} &= 7r_2^3 + \ hot + (2 + hot)u,
\end{aligned}$$

where, *hot* denote terms which are of order four and higher, results in a positive definite $r(x)$, and hence corresponds to a stabilizable system.

4 Concluding Remarks

Here we addressed the problem of asymptotically stabilizing homogeneous systems. We used Coleman's theorem as a guide. We derived a certain intrinsic polynomial that is closely linked with the stabilization problem, and show that the system is stabilizable if this polynomial is positive or negative definite.

There are still several unanswered questions regarding the stabilizability of homogeneous cubic systems. Primarily among them is whether this class is generically stabilizable whenever the degree is odd? If this isn't the case, is it possible to derive necessary and sufficient conditions for stabilizability of systems belonging to an appropriately chosen generic subset?

References

[1] C. Coleman. Asymptotic stability in 3-space, in *Contributions to the Theory of Nonlinear Oscillations*, Vol. V, (Cesari, LaSalle, and Lefschetz, eds.). *Annals of Mathematics Studies* **45**. Princeton, NJ: Princeton University Press, 1960.

[2] A. Andreini, A. Bacciotti, and G. Stefani. Global stabilizability of homogeneous vector fields of odd degree, *Syst. Cont. Lett.* **10** (1988), 251-256.

[3] J. Guckenheimer and P. Holmes. *Nonlinear Oscillations, Dynamical Systems, and Bifurcation of Vector Fields.* New York: Springer-Verlag, 1983.

[4] M. Kawski. Stabilization of nonlinear systems in the plane, *Systems and Control Letters* **12** (1989), 169-175.

[5] M. Kawski. Homogeneous stabilizing feedback laws, *Control-Theory and Advanced Technology* **4** (1990), 497-516.

[6] W.P. Dayawansa, C.F. Martin and S. Samelson. Asymptotic stabilization of a generic class of three-dimensional homogeneous quadratic systems, submitted to the *Systems and Control Letters.*

[7] W.P. Dayawansa and C.F. Martin. Some sufficient conditions for the asymptotic stabilizability of three dimensional homogeneous systems, *Proc. of the 18th IEEE Conference on Decision and Control, New York,* (December 1989), 1366-1370.

[8] W.P. Dayawansa, C. Martin and G. Knowles. Asymptotic stabilization of a class of smooth two dimensional systems, *SIAM J. on Control and Optimization* **28** (1990), 1321-1349.

[9] E. Brieskorn and H. Knorrer. Plane Algebraic Curves. Boston: Birkhäuser, 1986.

Department of Mathematics and Computer Science, State University of New York College at New Paltz, New Paltz, New York, 12561

Communicated by Clyde Martin