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Controllability of Semilinear Integrodifferential Systems in Banach Spaces*

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Abstract

Sufficient conditions for controllability of semilinear integrodifferential systems in a Banach space are established. The results are obtained using the asymptotic fixed-point theorem for k-set contractions.

Key words: controllability, integrodifferential systems

1 Introduction

Controllability of linear and nonlinear systems represented by ordinary differential equations has been studied by a number of authors. This concept has been extended to infinite dimensional systems in Banach spaces with bounded operators by Triggiani [12, 13]. Naito [8, 9] established the approximate controllability of semilinear control systems using fundamental assumptions on the systems components. Naito and Park [10] studied the same problem for delay Volterra control systems by using the Leray Schauder degree theorem. Yamanoto and Park [14] established necessary and sufficient conditions for the approximate controllability of a parabolic equation in a Banach space with uniformly bounded nonlinear term by estimating solutions to the nonlinear parabolic systems. Nakagiri and Yamamoto [6] gave a number of criteria for controllability and observability for retarded systems in general Banach spaces. Zhou [15] derived a set of sufficient conditions for the approximate controllability of semilinear abstract equation with a distributed control. Exact controllability of abstract semilinear equations has been studied by Lasiecka and Triggiani [5]. Conditions for approximate controllability for a nonlinear Volterra equation without any local restriction on reachable sets have been derived

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by Naito [9]. Recently Kwun et al. [4] investigated controllability and approximate controllability of delay Volterra systems by using the fixed-point theorem due to Nussbaum [11]. In this paper we shall also use the Nussbaum fixed-point technique in our study of the controllability of semilinear integrodifferential systems in Banach spaces.

2 Preliminaries

Consider a semilinear integrodifferential equation of the form

$$\begin{aligned} \dot{x}_t(\phi) + A(t)x_t(\phi) &= \int_{-\infty}^{t} f(t, s, x_s(\phi))ds + (Bu)(t), \\ & \quad t \in J = [0, T],\ x(t) = \phi(t),\ t \in (-\infty, 0] \qquad (2.1) \end{aligned}$$

where the state takes values $x(t)$ in the Banach space X and the control function u is given in $L^2(J, U)$, a Banach space of admissible control functions with U a Banach space. Here the linear operator $A(t)$ generates a strongly continuous evolution system $\{X(t,s)\}$ on X and is continuously initially observable [4]. Let C be a Banach space of all bounded uniformly continuous functions from $I = (-\infty, 0]$ to X endowed with the supremum norm

$$\|\phi\|_C = \sup\{\|\phi(\theta)\| : \theta \in I\}.$$

The nonlinear operator $f: R^+ \times R \times C \to X$ is compact and continuous. Then for the system (1) there exists a mild solution of the following form [3]

$$\begin{aligned} x_t(\phi) &= X(t,0)\phi(0) + \int_0^t X(t,s)\left[\int_{-\infty}^{s} f(s, \tau, x_\tau(\phi))d\tau\right] ds \\ &\quad + \int_0^t X(t,s)(Bu)(s)ds, \quad t \in J \qquad (2.2) \\ &\quad x(t) = \phi(t),\ t \in (-\infty, 0],\ \phi \in C. \end{aligned}$$

Definition System (2.1) is said to be *controllable* on the interval J if for every continuous initial function ϕ defined on $(-\infty, 0]$ and every $v \in X$, there exists a control $u \in L^2(J, U)$ such that the solution $x_t(\phi)$ of (1) satisfies $x_T(\phi) = v$. Throughout this work we let $\|z\|_2$ denote the L^2 norm on the space $L^2(J, M)$ for whatever Banach space M is associated with $z: J \to M$. We assume the following hypotheses (i) through (v):

(i) For the linear operator W from U to X defined by

$$Wu = \int_0^T X(T,s)Bu(s)ds$$

there is an invertible operator W^{-1} defined on $L^2(J,U)/\ker W$ and there exists a real valued function $L(\cdot)$ on J with positive values such that

$$\left\| \int_0^{\cdot} X(\cdot,s)Bu(s)ds \right\| \le L(\cdot)\|u\|_2$$

where $L(\cdot)$ is increasing and $L(0)=0$.

(ii) The evolution operator $\{X(t,s): 0 \le s \le t \le T\}$ is a compact mapping from X to itself, satisfying $X(t,s)x \in X \cap U$ for all $x \in X$, $0 \le s \le t$ and

$$\begin{aligned} \|X(t,s)x\|_X &\le p(t)\|x\|_X, \quad \|p\|_2 = c < \infty \\ \|X(t,s)x\|_U &\le q(t)\|x\|_X, \quad \|q\|_2 = d < \infty. \end{aligned}$$

(iii) The nonlinear function $f(t,s,y)$ is continuous and satisfies

$$\|f(t,s,x_s(\phi)) - f(t,s,y_s(\phi))\| \le g(t,s)\|x_s(\phi) - y_s(\phi)\|_C$$

where $g(\|x_s(\phi)\|, \|y_s(\phi)\|) = g(t,s)$ is continuous on $J \times J$, $g(t,s) \to 0$ as $t \to 0$ and $f(t,s,0)=0$ for $0 \le s \le t \le T$.

(iv) The function $Q(s) = \lim_{a\to\infty} \left[\int_{-a}^{0} f(s,\tau,\phi)\, d\tau \right]$ exists.

(v) A real number γ is chosen so that the following conditions hold

$$(c + L(t)d) \sup \left\{ \int_0^t g(\|\phi(s)\|, 0) ds : \|\phi\| \le \gamma \right\} \le k < 1,$$

$$c \sup \left\{ \int_0^t \int_0^s g(t,s) ds dt : 0 \le \phi(t), \psi(t) \le \gamma \right\} \le k < 1.$$

Now equation (2.2) can be written as

$$\begin{aligned} x_t(\phi) &= X(t,0)\phi(0) + \int_0^t X(t,s) \left[\int_0^s f(s,\tau,x_\tau(\phi))d\tau + Q(s) \right] ds \\ &\quad + \int_0^t X(t,s)(Bu)(s)ds, \quad t \in J \qquad (2.3) \\ &\quad x(t) = \phi(t),\ t \in (-\infty, 0],\ \phi \in C. \end{aligned}$$

To prove our main results we need the following result due to Nussbaum [6].

Theorem 1 *Suppose that S is a closed, bounded convex subset of a Banach space X, and that Φ_1, Φ_2 are continuous mappings from S into X such that*

(i) $(\Phi_1 + \Phi_2)S \subset S$,

(ii) $\|\Phi_1 x - \Phi_1 x'\| \leq k\|x - x'\|$ *for all* $x, x' \in S$ *where* k *is a constant and* $0 \leq k < 1$,

(iii) *the closure* $\overline{\Phi_2(S)}$ *is compact.*

Then the operator $\Phi_1 + \Phi_2$ *has a fixed point in* S.

3 Main Result

Theorem 2 *If the hypotheses* (i)–(v) *are satisfied, then the system* (2.1) *can be steered in the interval* J *from the initial state* ϕ *to any final state* v *satisfying*

$$\|v\|_U \leq \big[(1-k)\gamma - (c + Ld)(\|\phi\|_C + \|Q(s)\|_C)\big]/L.$$

Proof: Using the hypothesis (i), define the control

$$\begin{aligned} u(t) &= W^{-1}\bigg[v - X(T,0)\phi(0) - \int_0^T X(T,\rho)Q(\rho)d\rho \\ &\quad - \int_0^T X(T,\rho)\left\{\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau\right\}d\rho\bigg](t). \end{aligned}$$

This control is substituted into the equation (2.3) to define an operator Φ by

$$\Phi x_t(\phi) = \phi(t), \quad \text{for } t \in (-\infty, 0]$$

$$\begin{aligned} \Phi x_t(\phi) &= x(t,0)\phi(0) + \int_0^t X(t,s)Q(s)ds \\ &+ \int_0^t X(t,s)\left\{\int_0^s f(x,\tau,x_\tau(\phi))d\tau\right\}ds \\ &+ \int_0^t X(t,s)BW^{-1}\bigg[v - X(T,0)\phi(0) - \int_0^T X(T,\rho)Q(\rho)d\rho \\ &- \int_0^T X(T,\rho)\left\{\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau\right\}d\rho\bigg](s)ds, \quad t \in J. \end{aligned}$$

Clearly $\Phi x_T(\phi) = v$, which means that the control u steers the system from the initial state ϕ to v in time T provided we can obtain a fixed point of the nonlinear operator Φ. For that we define

$$\begin{aligned} \Phi_1 x_t(\phi) &= X(t,0)\phi(0) + \int_0^t X(t,\rho)Q(\rho)d\rho \\ &\quad + \int_0^t X(t,\rho)\left\{\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau\right\}d\rho \end{aligned}$$

and

$$\begin{aligned}\Phi_2 x_t(\phi) &= \int_0^t X(t,s)BW^{-1}\Big[v - X(T,0)\phi(0) - \int_0^T X(T,\rho)Q(\rho)d\rho \\ &\quad - \int_0^T X(T,\rho)\left\{\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau\right\}d\rho\Big](s)ds.\end{aligned}$$

We can apply Theorem 1 with

$$S = \{x_t(\phi)(\cdot) \in C \colon \|x_t(\phi)\| \le \gamma\}.$$

Then the set S is closed, bounded and convex. From the definition, we have

$$\Phi x_t(\phi)(0) = \Phi_1 x_t(\phi)(0) + \Phi_2 x_t(\phi)(0).$$

Thus, for any $x_t(\phi)(\cdot) \in S$,

$$\begin{aligned}\|\Phi x_t(\phi)(\theta)\|_X &= \|\Phi x_{t+\theta}(\phi)(0)\| \le \|X(t+\theta,0)\phi(0)\| \\ &+ \int_0^{t+\theta} \|X(t+\theta,s)Q(s)\|ds \\ &+ \int_0^{t+\theta} \left\|X(t+\theta,s)\left\{\int_0^s f(s,\tau,x_\tau(\phi))d\tau\right\}\right\| ds \\ &+ \int_0^{t+\theta} \Big\|X(t+\theta,s)BW^{-1}\Big[v - X(T,0)\phi(0) - \int_0^T X(T,\rho)Q(\rho)d\rho \\ &- \int_0^T X(T,\rho)\left\{\int_0^\rho f(\rho,\tau,x(\phi))d\tau\right\}d\rho\Big](s)\Big\|ds \\ &\le (c+Ld)[\|\phi\|_C + \|Q(s)\|_C] + L\|v\|_2 + k\gamma \\ &\le (1-k)\gamma + k\gamma = \gamma, \quad -h \le \theta \le 0.\end{aligned}$$

Hence

$$\sup\big\{\|\Phi x_t(\phi)(\theta)\|_X \colon -h \le \theta \le 0\big\} = \|\Phi x_t(\phi)\|_C \le \gamma.$$

Hence $\Phi_1 x_t(\phi)(0) + \Phi_2 x_t(\phi)(0) \in S$ for all $x_t(\phi) \in S$. Which means that part (i) of Theorem 1 is satisfied. To prove the part (ii) of Theorem 1, consider two functions $x_t(\phi)$, $\hat{x}_t(\phi) \in S$, then

$$\begin{aligned}&\|\Phi_1 x_t(\phi)(\theta) - \Phi_1 \hat{x}_t(\phi)(\theta)\|_X \\ &\quad\le \left\|\int_0^{t+\theta} X(t+\theta,s)\left[\int_0^s |f(s,\tau,x_\tau(\phi)) - f(s,\tau,\hat{x}_t(\phi))|d\tau\right]ds\right\|_X \\ &\quad\le c\int_0^t\int_0^s g(s,t)dsdt(\|x_\tau(\phi) - \hat{x}_t(\phi)\|_C.\end{aligned}$$

Consequently,

$$\|\Phi_1 x_t(\phi) - \Phi_1 \hat{x}_t(\phi)\|_C \le k\|x_\tau(\phi) - \hat{x}_\tau(\phi)\|_C, \quad 0 \le k < 1.$$

Finally we must show that Φ_1 and Φ_2 are completely continuous. For this consider

$$\begin{aligned}
&\|\Phi_1(x_t(\phi)+\eta)(\theta)-\Phi_1 x_t(\phi)(\theta)\|_X \\
&\quad = \|\Phi_1(x_{t+\theta}(\phi)+\eta)(0)-\Phi_1 x_{t+\theta}(\phi)(0)\|_X \\
&\quad = \left\|\int_0^{t+\theta} X(t+\theta,s)\left[\int_0^s \{f(s,\tau,x_\tau(\phi)+\eta)-f(s,\tau,x_\tau(\phi))\}d\tau\right]ds\right\|_X \\
&\quad \leq \|p\|_2\left(\int_0^t\int_0^s g(s,\tau)d\tau ds\right)\|\eta\|_C.
\end{aligned}$$

Hence

$$\begin{aligned}
&\sup\{\|\Phi_1(x_t(\phi)+\eta)(\theta)-\Phi_1 x_t(\phi)(\theta)\|_X : -h\leq\theta\leq 0\} \\
&\quad = \|\Phi_1(x_t(\phi)+\eta)-\Phi_1 x_t(\phi)\|_C \\
&\quad \leq \|p\|_2\left(\int_0^t\int_0^s g(s,\tau)d\tau ds\right)\|\eta\|_C \to 0 \quad \text{as } \eta\to 0.
\end{aligned}$$

Similarly

$$\begin{aligned}
&\sup\{\|\Phi_2(x_t(\phi)+\eta')\theta)-\Phi_2 x_t(\phi)(\theta)\|_X : -h\leq\theta\leq 0\} \\
&\quad = \|\Phi_2(x_t(\phi)+\eta')-\Phi_2 x_t(\phi)\|_C \\
&\quad \leq L\|q\|_2\left(\int_0^T\int_0^s g(s,\tau)d\tau ds\right)\|\eta'\|_C \to \quad \text{as } \eta'\to 0.
\end{aligned}$$

Thus Φ_1 and Φ_2 are continuous. Using the Arzela–Ascoli theorem we show that Φ_2 maps S into a precompact subset of S. To show this define

$$\begin{aligned}
\Phi_2 x_{t-\varepsilon}(\phi)(\theta) = &\int_0^{t+\theta-\varepsilon} X(t+\theta,s)BW^{-1}\Big[v-X(T,0)\phi(0) \\
&-\int_0^T X(T,\rho)Q(\rho)d\rho \\
&-\int_0^T X(T,\rho)\left\{\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau\right\}d\rho\Big](s)ds
\end{aligned}$$

for all $x_t(\phi)\in S$. Thus, we have

$$\begin{aligned}
&\Phi_2 x_{t-\varepsilon}(\phi)(\theta) = X(t+\theta,t+\theta-\varepsilon)\cdot \\
&\quad \cdot\int_0^{t+\theta-\varepsilon} X(t+\theta-\varepsilon,s)BW^{-1}\Big[v-X(T,0)(\phi)(0) \\
&\quad -\int_0^T X(T,\rho)Q(\rho)d\rho \\
&\quad -\int_0^T X(T,\rho)\left\{\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau\right\}d\rho\Big](s)ds.
\end{aligned}$$

Since $X(t+\theta, t+\theta-\varepsilon)$ is a compact operator, the set

$$K_2[x_t(\phi)(\theta)] = \{\Phi_2 x_{t-\varepsilon}(\phi)(\theta) \colon x_t(\phi) \in S\}$$

is precompact. Also

$$\begin{aligned}
&\|\Phi_2 x_t(\phi)(\theta) - \Phi_2 x_{t-\varepsilon}(\phi)(\theta)\|_X \\
&\quad = \left\| \int_{t+\theta-\varepsilon}^{t+\theta} X(t+\theta, s) B W^{-1} \Bigg[v - X(T,0)(\phi)(0) \right. \\
&\qquad - \int_0^T X(T,\rho) Q(\rho)\, d\rho \\
&\qquad \left. - \int_0^T X(T,\rho) \left\{ \int_0^\rho f(\rho, \tau, x_\tau(\phi)) d\tau \right\} d\rho \Bigg](s) ds \right\|_X \\
&\quad \leq \; L(\varepsilon) \Bigg\{ \|v\|_2 + d \Bigg[\|\phi(0)\|_C + \|Q(s)\|_C \\
&\qquad + \int_0^T \int_0^s g(\rho, \tau) d\tau ds \Bigg] \|x_\tau(\phi)\|_C \Bigg\} \to 0
\end{aligned}$$

as $\varepsilon \to 0$. Hence

$$\begin{aligned}
&\sup\{\|\Phi_2 x_t(\phi)(\theta) - \Phi_2 x_{t-\varepsilon}(\phi)(\theta)\|_X \colon -h \leq \theta \leq 0\} \\
&\quad = \; \|\Phi_2 x_{t+\theta}(\phi) - \Phi_2 x_{t+\theta-\varepsilon}(\phi)\|_C \\
&\quad \leq \; L(\varepsilon) \Bigg\{ \|v\|_2 + d \Bigg[\|\phi(0)\|_C + \|Q(s)\|_C \\
&\qquad + \left(\int_0^T \int_0^s g(s, \tau) d\tau ds \right) \Bigg] \|x_\tau(\phi)\|_C \Bigg\} \to 0 \quad \text{as } \varepsilon \to 0.
\end{aligned}$$

Thus there are compact sets arbitrarily close to the set

$$K_2[x_t(\phi)(\theta)] = \{\Phi_2 x_t(\phi)(\theta) \colon x_t(\phi) \in S\}$$

and therefore $K_2[x_t(\phi)(\theta)]$ is precompact. We next show that Φ_2 maps the functions in S into an equicontinuous family of functions. For equicontinuous from the left we take $t > \varepsilon > t' > 0$, then

$$\begin{aligned}
&\|\Phi_2 x_t(\phi)(\theta) - \Phi_2 x_{t-t'}(\phi)(\theta)\|_X \\
&= \Bigg\| \int_0^{t+\theta} X(t+\theta, s) B W^{-1} \Bigg[v - X(T,0)(\phi)(0) \\
&\quad - \int_0^T X(T,\rho) Q(\rho) d\rho - \int_0^T X(T,\rho) \left\{ \int_0^\rho f(\rho, \tau, x_\tau(\phi))\, d\tau \right\} d\rho \Bigg](s) ds \\
&\quad - \int_0^{t-t'+\theta} X(t-t'+\theta, s) B W^{-1} \Bigg[v - X(T,0)\phi(0) - \int_0^T X(T,\rho) Q(\rho) d\rho \\
&\quad - \int_0^T X(T,\rho) \left\{ \int_0^\rho f(\rho, \tau, x_\tau(\phi)) d\tau \right\} d\rho \Bigg](s) ds \Bigg\|_X
\end{aligned}$$

$$
\begin{aligned}
&\leq \Bigg\| \int_0^{t+\theta-\varepsilon} X(t+\theta,s)BW^{-1}\Bigg[v - X(T,0)\phi(0) - \int_0^T X(T,\rho)Q(\rho)d\rho \\
&\quad - \int_0^T X(T,\rho)\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau d\rho\Bigg](s)ds \\
&\quad - \int_0^{t+\theta-\varepsilon} X(t-t'+\theta,s)BW^{-1}\Bigg[v - X(T,0)\phi(0) - \int_0^T X(T,\rho)Q(\rho)d\rho \\
&\quad - \int_0^T X(T,\rho)\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau d\rho\Bigg](s)ds\|_X \\
&\quad + \Bigg\| \int_{t+\theta-\varepsilon}^{t+\theta} X(t+\theta,s)BW^{-1}\Bigg[v - X(T,0)\phi(0) - \int_0^T X(T,\rho)Q(\rho)d\rho \\
&\quad - \int_0^T X(T,\rho)\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau d\rho\Bigg](s)ds\Bigg\|_X \\
&\quad + \Bigg\| \int_{t+\theta-\varepsilon}^{t-t'+\theta} X(t-t'+\theta,s)BW^{-1}\Bigg[v - X(T,0)\phi(0) - \int_0^T X(T,\rho)Q(\rho)d\rho \\
&\quad - \int_0^T X(T,\rho)\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau d\rho\Bigg](s)ds\Bigg\|_X \\
&\leq \|X(t+\theta,t-t'+\theta) - I\| \int_0^{t+\theta-\varepsilon} \Bigg\| X(t-t'+\theta,s)BW^{-1}\Bigg[v - X(T,0)\phi(0) \\
&\quad - \int_0^T X(T,\rho)Q(\rho)d\rho - \int_0^T X(T,\rho)\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau d\rho\Bigg](s)\Bigg\|_X ds \\
&\quad + \int_{t+\theta-\varepsilon}^{t+\theta} \Bigg\| X(t+\theta,s)BW^{-1}\Bigg[v - X(T,0)\phi(0) - \int_0^T X(T,\rho)Q(\rho)d\rho \\
&\quad - \int_0^T X(T,\rho)\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau d\rho\Bigg](s)\Bigg\|_X ds \\
&\quad + \Bigg\| \int_{t+\theta-\varepsilon}^{t-t'+\theta} X(t-t'+\theta,s)BW^{-1}\Bigg[v - X(T,0)\phi(0) \\
&\quad - \int_0^T X(T,\rho)Q(\rho)d\rho - \int_0^T X(T,\rho)\int_0^\rho f(\rho,\tau,x_\tau(\phi))d\tau d\rho\Bigg](s)\Bigg\|_X ds \\
&\leq \|X(t+\theta,t-t'+\theta) - I\|L(t+\theta-\varepsilon)\|u\|_X \\
&\quad + L(\varepsilon)\|u\|_X + L(\varepsilon-t')\|u\|_X \to 0
\end{aligned}
$$

as $\varepsilon \to 0$, since $L(t) \to 0$ as $t \to 0$ and $X(t,s)$ is continuous in (s,t). Thus we have

$$
\begin{aligned}
&\sup\{\|\Phi_2 x_t(\phi)(\theta) - \Phi_2 x_{t-t'}(\phi)(\theta)\|_X : -h \leq \theta \leq 0\} \\
&\quad = \|\Phi_2 x_t(\phi) - \Phi_2 x_{t-t'}(\phi)\| \to 0 \quad \text{as } t' \to 0.
\end{aligned}
$$

The equicontinuity from the right is similar. Hence by Theorem 1, Φ has a fixed point.

4 Example

Let Ω be a domain in R^3 with smooth boundary. Consider the integrodifferential equation

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} + \int_{-\infty}^{t} g(t-s)h(\nabla y)ds + u(t) \tag{4.1}$$

$$\begin{array}{ll} y(x,t) = 0, & (x,t) \in \delta\Omega \times R \\ y(x,t) = \phi(x,t), & t \in (-\infty, 0] \end{array}$$

where the operator $A = \delta^2/\delta x^2 \colon H^2(0,1) \cap H_0^1(0,1) \to H_0^1(0,1)$ generates a strongly continuous compact semigroup [see 1, 2] given by

$$X(t)y = \sum_{i=1}^{\infty} \exp(-n^2\pi^2 t)(y, \phi_n)\phi_n,$$

where $\phi_n = (\sqrt{2})\sin(n\pi x)$, $g(\cdot)\colon R^+ \to R$ is continuous and $g(t-s) \to 0$ as $t \to s$, the nonlinearity $h\colon R^3 \to R$ vanishes at zero and has the property that there exist a $c_0 > 0$ such that

$$|h(u) - h(v)| \leq c_0 \sum_{i=1}^{3} |u_i - v_i|, \quad \text{for } u, v \in L^2(J, L^2(0,1)).$$

It is known [1] that the linear version for (4.1)

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} + u(t)$$

is controllable in $H_0^1(0,1)$ where

$$H_0^1(0,1) = \left\{ z \in L^2(0,1)\colon \sum_n n^2 |\langle z, \phi_n \rangle|^2 < \infty \right\}.$$

Further, the constants c, d and L are finite. Let

$$r = \sup \left\{ \int_0^t g(t-s)ds \colon 0 \leq t \leq 1 \right\}$$

be such that $r < k/(c + Ld) \leq 1$. Hence all the conditions (ii)–(v) are satisfied, by Theorem 2 the system (4.1) is controllable on $H_0^1(0,1)$.

References

[1] E.N. Chukwu and S.M. Lenhart Controllability questions for nonlinear systems in abstract spaces, *J. Optim. Theory & Appl.* **68** (1991), 437–462.

[2] R.F. Curtain and A.J. Pritchard. *Infinite Dimensional Linear Systems Theory.* New York: Springer-Verlag, 1978.

[3] N.E. Fitzgibbon. Asymptotic behavior of solutions to a class of Volterra integrodifferential equations, *J. Math. Anal. Appl.* **146** (1990), 241–253.

[4] Y.C. Kwun, J.Y. Park, and J.W. Ryu. Approximate controllability and controllability for delay Volterra system, *Bull. Korean Math. Soc.* **28** (1991), 131–145.

[5] I. Lasiecka and R. Triggiani. Exact controllability of semilinear abstract systems with application to waves and plates boundary control problems, *Appl. Math. Optim.* **23** (1991), 109–154.

[6] S. Nakagiri and M. Yamamoto. Controllability and observability of linear retarded systems in Banach spaces, *Int. J. Control* **49** (1989), 1489–1504.

[7] K. Naito. Controllability of semilinear control systems dominated by the linear part, *SIAM J. Control Optim.* **25** (1987), 715–722.

[8] K. Naito. Approximate controllability for trajectories of semilinear control systems, *J. Optim. Theory Appl.* **60** (1989), 57–65.

[9] K. Naito. On controllability for a nonlinear Volterra equation, *Nonlinear Analysis, Theory, Methods & Appl.* **18** (1992), 99–108.

[10] K. Naito and J.Y. Park. Approximate controllability for trajectories of a delay Volterra control system, *J. Optim. Theory Appl.* **61** (1989), 271–279.

[11] R.P. Nussbaum. The fixed point index and asymptotic fixed point theorems for k-set contractions, *Bull. Amer. Math. Soc.* **75** (1969), 490–495.

[12] R. Triggiani. Controllability and observability in Banach spaces with bounded operators, *SIAM J. Control* **3** (1975), 462–491.

[13] R. Triggiani. Extensions of rank conditions for controllability and observability to Banach space and unbounded operators, *SIAM J. Control and Optimization* **14** (1976), 313–338.

[14] M. Yamamoto and J.Y. Park. Controllability of parabolic equations with uniformly bounded nonlinear terms, *J. Optim. Theory Appl.* **66** (1990), 515–532.

[15] H.X. Zhou. Approximate controllability for a class of semilinear abstract equations, *SIAM J. Control & Optim.* **21** (1983), 551–565.

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